

Opportunistic Adaptive Relaying in Cognitive Radio Networks

Wael Jaafar
École Polytechnique de Montréal
Department of Electrical Engineering
Montreal, Canada
Email: wael.jaafar@polymtl.ca

Wessam Ajib
Université du Québec à Montréal
Department of Computer Science
Montreal, Canada
Email: ajib.wessam@uqam.ca

David Haccoun
École Polytechnique de Montréal
Department of Electrical Engineering
Montreal, Canada
Email: david.haccoun@polymtl.ca

Abstract—Combining cognitive radio technology with user cooperation could be advantageous to both primary and secondary transmissions. In this paper, we propose a first relaying scheme for cognitive radio networks (called “Adaptive relaying scheme 1”), where one relay node can assist the primary or the secondary transmission with the objective of improving the outage probability of the secondary transmission with respect to a primary outage probability threshold. Upper bound expressions of the secondary outage probability using the proposed scheme are derived over Rayleigh fading channels. Numerical and simulation results show that the secondary outage probability using the proposed scheme is lower than that of other relaying schemes. Then, we extend the proposed scheme to the case where the relay node has the ability to decode both the primary and secondary signals and also can assist simultaneously both transmissions. Simulations show the performance improvement that can be obtained due to this extension in terms of secondary outage probability.

I. INTRODUCTION

In order to overcome the problems related to the rigid allocation of spectrum bands to few licensed operators and the under-utilization of these bands, Cognitive Radio (CR) technology has evolved in wireless communications for allowing unlicensed secondary users (SUs) to access licensed parts of the spectrum without harmfully interfering with the transmissions of the licensed primary users (PUs) taking place in the same spectrum band [1]-[2]. In underlay spectrum sharing mode, SUs are allowed to transmit simultaneously with PUs if they tune their transmission parameters (such as transmit power) to be harmless to primary transmissions.

Meanwhile, user cooperation has been recognized as an interesting technique that allows to achieve increased diversity order when one or several relay nodes assist the transmission [3]-[4].

Consequently, combining user cooperation and cognitive radio has recently attracted attention to improve both the spectrum utilization and the transmission performances. User cooperation has been applied for the primary transmission when a cognitive secondary transmitter acts as a relay [5]. By doing so, the primary outage probability is improved, while SUs have more opportunities to access the licensed spectrum bands and hence secondary transmission performances can be also improved. In [6], an adaptive user cooperation scheme with best-relay selection is proposed in multiple-relay

Cognitive Radio Networks (CRNs) to improve the secondary outage performance while satisfying a primary outage probability threshold. By letting the “best” CR relay assist the secondary transmission, the secondary outage probability can be considerably reduced. In [7], secondary transmissions are assisted by a group of CR relay nodes located at different positions. The outage probability was investigated when all the relays forward simultaneously their received signals. The system achieves full diversity when the number of cooperating relay nodes is adequately selected. In [8], we proposed a new relaying scheme for CRNs where a CR relay node is able to assist simultaneously the primary and secondary transmissions. It has been shown that for certain relay’s position, assisting simultaneously both transmissions provides better outage performance than assisting only the primary or the secondary transmission. However, the proposed scheme is greedy on the relay’s transmit power.

Even though, the previous works have investigated the utilization of user cooperation in CRNs for assisting the primary transmissions, the secondary transmissions or both, no work has investigated adaptive relaying schemes where the relay node decides independently when and which communication to assist. Consequently, we propose in this paper a novel opportunistic adaptive relaying scheme, where the CR relay node is able to decide when to cooperate or not, and in case of cooperation, whom to cooperate with (primary transmission or secondary transmission) depending on the channel states. We propose to extend the adaptive relaying scheme to the case where the relay node can also cooperate simultaneously with both transmissions.

The paper is organized as follows. Section II presents the system model. In section III, we describe the two adaptive relaying schemes and we provide analytically the secondary outage probability for the first adaptive relaying scheme. Section IV shows and discusses the numerical and simulation results and a conclusion closes the paper in section V.

II. SYSTEM MODEL

We assume a CRN where one primary transmitter (PT) transmits data to a primary destination (PD) and a secondary transmitter (ST) communicates with a secondary destination (SD) over the same frequency band (Fig. 1). We assume a

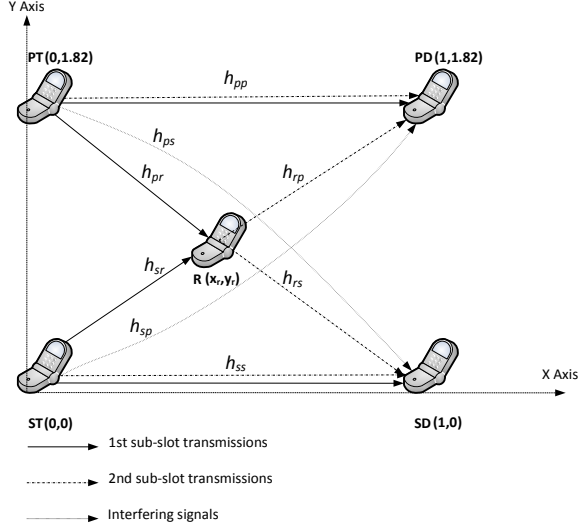


Fig. 1. The Cognitive Radio Network

decode-and-forward secondary CR relay node (R) that can assist the primary or the secondary transmission in order to increase the secondary access to the licensed spectrum bands with respect to a certain primary outage probability threshold. In the extension, we assume that the relay is able to assist both transmissions.

We assume that PT and ST transmit their signals x_p and x_s (where $E\{|x_p|^2\} = E\{|x_s|^2\} = 1$) with powers P_p and P_s respectively in order to achieve data rates R_p and R_s respectively. We assume also that R uses transmit power $P_r \leq P_r^{max}$, where P_r^{max} is the maximal relay transmit power. We assume that the channels are submitted to Rayleigh fading and path loss attenuation and are stationary during a time-slot (time slot = 1st + 2nd sub-slots).

Following Fig. 1, the received signals during the first sub-slot are expressed by:

$$y_a(1) = \sqrt{P_p}h_{pa}x_p + \sqrt{P_s}h_{sa}x_s + n_a, \quad (1)$$

where $a = p, s$ or r (p, s and r denote primary, secondary and relay node resp.), h_{ba} ($b = p$ or s) is the channel gain between nodes b and a having variance $\sigma_{ba}^2 = d_{ba}^{-\beta}$, d_{ba} is the distance between b and a , β is the path-loss exponent, and where n_a is the additive white gaussian noise with zero mean and variance N_0 received at a . We assume a fixed P_p and that P_s is calculated with respect to the primary outage probability threshold denoted ε . P_s is given similarly to [6] by:

$$P_s = \frac{2P_p\sigma_{pp}^2}{\Lambda_p\sigma_{sp}^2}\rho^+, \quad (2)$$

where $\rho^+ = \max(0, \rho)$, $\rho = \frac{e^{-\frac{\Lambda_p}{2\gamma_p\sigma_{pp}^2}}}{1-\varepsilon} - 1$, $\Lambda_p = 2^{2R_p} - 1$ and $\gamma_p = P_p/N_0$. ST calculates P_s assuming that PT repeats the same signal over the two sub-slots with the same transmit power P_p .

III. ADAPTIVE RELAYING SCHEMES

A. Description of adaptive relaying scheme 1

This novel scheme aims to exploit efficiently the relay position, the acquired information and the propagation environment conditions. We define by $a_0 = \frac{\gamma_s|h_{ss}|^2}{\gamma_p|h_{ps}|^2+1}$, $a_p = \frac{\gamma_s|h_{ss}|^2}{\gamma_r^{(p)}|h_{rs}|^2+1}$, $a_s = \frac{\gamma_r^{(s)}|h_{rs}|^2}{\gamma_p|h_{ps}|^2+1}$ where $\gamma_a = P_a/N_0$ ($a = p$ or s) and $\gamma_r^{(i)}$ is the transmit power of the relay node when assisting transmission i . We also define $E_i = \{a_i = \max\{a_p, a_s, a_0\}\}$ the opportunism condition ($i = p, s$ or 0). At the first sub-slot, R attempts to decode x_p or x_s and then, a relaying procedure is chosen depending on the value of D defined by:

$$\begin{aligned} \text{If } A_p \cap \{\{\bar{A}_s \cap (a_p > a_0)\} \cup \{A_s \cap E_p\}\}, \text{ then } D &= 1, \\ \text{If } A_s \cap \{\{\bar{A}_p \cap (a_s > a_0)\} \cup \{A_p \cap E_s\}\}, \text{ then } D &= 2, \\ \text{Otherwise } D &= 0, \end{aligned} \quad (3)$$

where \bar{A} is the complement of A , and

$$A_p = \left\{ \frac{1}{2} \log_2 \left(1 + \frac{\gamma_p|h_{pr}|^2}{\gamma_s|h_{sr}|^2+1} \right) \geq R_p \right\}, \quad (4)$$

$$A_s = \left\{ \frac{1}{2} \log_2 \left(1 + \frac{\gamma_s|h_{sr}|^2}{\gamma_p|h_{pr}|^2+1} \right) \geq R_s \right\}. \quad (5)$$

The comparison of $(a_i : i = p, s \text{ or } 0)$ indicates which relaying would improve better the secondary outage probability.

The different cases are detailed below.

1) *R assists the primary transmission ($D = 1$):* This case occurs either when (i) R succeeds to decode the primary signal but not the secondary signal and when relaying the primary signal provides lower secondary outage probability than the repetition (i.e., $a_p > a_0$) or (ii) R succeeds to decode both the primary and secondary signals and assisting the primary transmission provides the lowest $P_{out,sec}$ (i.e., E_p). Hence, when the relay is able to decode the primary signal and the best choice is to assist the primary transmission, then $D = 1$. Consequently, the received signals at PD and SD, on the second sub-slot, are respectively given by:

$$y_a(2|D=1) = \sqrt{P_r^{(p)}}h_{ra}x_p + \sqrt{P_s}h_{sa}x_s + n_a. \quad (6)$$

After normalizing the noise variances and combining the signals received in the two sub-slots (given by (1) and (6)) with Maximum Ratio Combining (MRC) [5], the Signal-to-Interference-plus-Noise-Ratio (SINR) at PD is:

$$\text{SINR}_p(D=1) = \frac{\gamma_p|h_{pp}|^2}{\gamma_s|h_{sp}|^2+1} + \frac{\gamma_r^{(p)}|h_{rp}|^2}{\gamma_s|h_{sp}|^2+1}, \quad (7)$$

while SINR_s at SD is given by:

$$\text{SINR}_s(D=1) = \frac{\gamma_s|h_{ss}|^2}{\gamma_p|h_{ps}|^2+1} + \frac{\gamma_r^{(p)}|h_{rs}|^2}{\gamma_p|h_{ps}|^2+1}. \quad (8)$$

2) *R assists the secondary transmission ($D = 2$):* This second case occurs when (i) R succeeds to decode only the

secondary signal and when relaying the secondary signal is beneficial to the secondary transmission (i.e., $a_s > a_0$) or (ii) R succeeds to decode both signals and assisting the secondary transmission provides the lowest $P_{out,sec}$ (i.e., E_s). In this case, R assists the secondary transmission and $D = 2$. The received SINR at PD and that at SD are expressed as (8) and (7) respectively, where indexes p and s are inverted.

3) *R does not assist the transmissions ($D = 0$):* When the relay is not able to decode the signals or when relaying is not beneficial to $P_{out,sec}$, the relay does not participate in the transmissions. In this case, we assume that the primary and secondary transmitters retransmit the same signals. Accordingly, the received SINR at SD is given by (eq.(7), [8]) and that at PD by inverting indexes p and s in (eq.(7), [8]).

B. Outage Probability Analysis

In this scheme, any of three cases can happen. We start by calculating the probability of occurrence of each one.

An upper bound on the probability of occurrence of $D = 1$ is given by:

$$\begin{aligned}
 P(D = 1) &= \frac{\tilde{\gamma}_{ps}}{\tilde{\gamma}_{ps} + \tilde{\gamma}_{rs}^{(p)}} \times \frac{\tilde{\gamma}_{pr} e^{-\frac{\Lambda_p}{\tilde{\gamma}_{pr}} - \frac{\Lambda_s(1+\Lambda_p)}{(1-\Lambda_p\Lambda_s)\tilde{\gamma}_{sr}}}}{\tilde{\gamma}_{pr} + \Lambda_p \tilde{\gamma}_{sr}} \\
 &+ \frac{\tilde{\gamma}_{ps}}{\tilde{\gamma}_{ps} + \tilde{\gamma}_{rs}^{(p)}} \left(1 - \frac{\tilde{\gamma}_{sr} e^{-\frac{\Lambda_s}{\tilde{\gamma}_{sr}}}}{\tilde{\gamma}_{sr} + \Lambda_s \tilde{\gamma}_{pr}} \right) \\
 &\times \left(1 - e^{-\frac{\Lambda_s(1+\Lambda_p)}{(1-\Lambda_p\Lambda_s)\tilde{\gamma}_{sr}}} \right) - \phi_i \left(-\frac{1}{\tilde{\gamma}_{ps}} - \frac{\tilde{\gamma}_{rs}^{(s)}}{\tilde{\gamma}_{ss}\tilde{\gamma}_{ps}} \right) \\
 &\times \frac{\tilde{\gamma}_{ps}}{\tilde{\gamma}_{ps} + \tilde{\gamma}_{rs}^{(p)}} \left(\frac{1 + \tilde{\gamma}_{ps}}{\tilde{\gamma}_{ss}} \right) e^{\frac{1}{\tilde{\gamma}_{ps}} + \frac{\tilde{\gamma}_{rs}^{(s)}}{\tilde{\gamma}_{ss}\tilde{\gamma}_{ps}}} \\
 &\times \frac{\tilde{\gamma}_{sr} e^{-\frac{\Lambda_s}{\tilde{\gamma}_{sr}} - \frac{\Lambda_p(1+\Lambda_s)}{(1-\Lambda_p\Lambda_s)\tilde{\gamma}_{pr}}}}{\tilde{\gamma}_{sr} + \Lambda_s \tilde{\gamma}_{pr}}, \quad (9)
 \end{aligned}$$

where $\tilde{\gamma}_{ab} = \gamma_a \sigma_{ab}^2$, $\tilde{\gamma}_{rb}^{(i)} = \gamma_r^{(i)} \sigma_{rb}^2$ ($a = p, s$ or r and $b = p, s$ or r) and $\phi_i(x) = \int_{-\infty}^x \frac{e^t}{t} dt$.

Proof: See Appendix A. ■

By following similar calculations, we can obtain $P(D = 2)$. Finally, $P(D = 0) = 1 - \sum_{i=1}^2 P(D = i)$.

We next present the conditional primary and conditional secondary outage probabilities for each case:

- $D = 1$

The conditional primary outage probability is given by:

$$\begin{aligned}
 P_{pri}(out.|D = 1) &= P(SINR_p(D = 1) < \Lambda_p) \\
 &= P(\omega < \Lambda_p + \Lambda_p \omega_1 - \omega_2) \quad (10)
 \end{aligned}$$

where $\Lambda_a = 2^{2R_a} - 1$ ($a = p$ or s), $\omega = \gamma_r^{(p)} |h_{rp}|^2$, $\omega_1 = \gamma_s |h_{sp}|^2$ and $\omega_2 = \gamma_p |h_{pp}|^2$. We shall make use of the following Lemma.

Lemma 1. *The exact closed-form expression of the conditional primary outage probability is expressed by:*

$$P_{pri}(out.|D = 1) = \lambda_1 + \lambda_2, \quad (11)$$

where

$$\lambda_1 = \frac{\tilde{\gamma}_{sp}^2 \Lambda_p^2 + \tilde{\gamma}_{rp}^{(p)} \tilde{\gamma}_{sp} \Lambda_p \left(1 - e^{-\frac{\Lambda_p}{\tilde{\gamma}_{rp}^{(p)}}} \right)}{(\tilde{\gamma}_{pp} + \Lambda_p \tilde{\gamma}_{sp})(\tilde{\gamma}_{rp}^{(p)} + \Lambda_p \tilde{\gamma}_{sp})}, \quad (12)$$

$$\begin{aligned}
 \lambda_2 &= \frac{\tilde{\gamma}_{pp}^2 \left(1 - e^{-\frac{\Lambda_p}{\tilde{\gamma}_{pp}}} \right) - \tilde{\gamma}_{pp} \tilde{\gamma}_{rp}^{(p)} \left(1 - e^{-\frac{\Lambda_p}{\tilde{\gamma}_{rp}^{(p)}}} \right)}{(\tilde{\gamma}_{pp} + \Lambda_p \tilde{\gamma}_{sp})(\tilde{\gamma}_{rp}^{(p)} - \tilde{\gamma}_{pp})}, \\
 \forall \quad \tilde{\gamma}_{pp} &\neq \tilde{\gamma}_{rp}^{(p)}. \quad (13)
 \end{aligned}$$

Proof: See Appendix B. ■

Lemma 2. *An upper bound on the conditional secondary outage probability is given by:*

$$\begin{aligned}
 P_{sec}(out.|D = 1) &= \left(1 - e^{-\frac{\Lambda_s}{\tilde{\gamma}_{ss}}} \left(1 + \frac{\ln \left(1 + \frac{\Lambda_s \tilde{\gamma}_{ps}}{\tilde{\gamma}_{ss}} \right)}{\tilde{\gamma}_{ps}} \right) \right) \\
 &\times \left(\tilde{\gamma}_{rs}^{(p)} + 1 \right) = \varphi \times \left(\tilde{\gamma}_{rs}^{(p)} + 1 \right). \quad (14)
 \end{aligned}$$

Proof: See Appendix C. ■

By assisting the primary transmission, this relaying procedure aims to reduce the interference caused to the secondary transmission with respect to the primary outage threshold ε . For that purpose, R should control its transmit power P_r to be as low as possible. This value of P_r , denoted $P_{r,num}^{(p)}$, is evaluated numerically by solving $P_{pri}(out.|D = 1) = \varepsilon$. If $P_{r,num}^{(p)} > P_r^{max}$, then relaying is not beneficial and $D = 0$.

- $D = 2$

Due to the similarity of the outage probability analysis of this relaying procedure to the first one, it is not given in details. However, by inverting indexes p and s and indexes pri and sec in equations (10)-(14), we obtain an accurate outage probability analysis. The relay transmit power should also be calculated such that $P_{pri}(out.|D = 2) = \varepsilon$. The relay transmit power is given by $\gamma_{r,num}^{(s)} = \frac{\varepsilon/\varphi' - 1}{\sigma_{rp}^2}$, where

$$\varphi' = \left(1 - e^{-\frac{\Lambda_p}{\tilde{\gamma}_{pp}}} \left(1 + \frac{\ln \left(1 + \frac{\Lambda_p \tilde{\gamma}_{sp}}{\tilde{\gamma}_{pp}} \right)}{\tilde{\gamma}_{sp}} \right) \right).$$

Then, $\gamma_r^{(s)} = \min \left(\gamma_{r,num}^{(s)}, \gamma_r^{max} \right)$.

- $D = 0$

The outage probability analysis of this case is presented in [8]. The conditional secondary outage probability is given by (eq.(16), [8]), while $P_{pri}(out.|D = 0)$ is obtained by simply inverting indexes p and s in (eq.(16), [8]).

Finally, upper bounds on the primary and secondary outage probabilities are given by:

$$P_{out,c} = \sum_{i=0}^2 P(D = i) P_c(out.|D = i), \quad (15)$$

where $c = pri$ or sec . The obtained expressions are upper bounds since some of the conditional outage probabilities calculated are upper bounds.

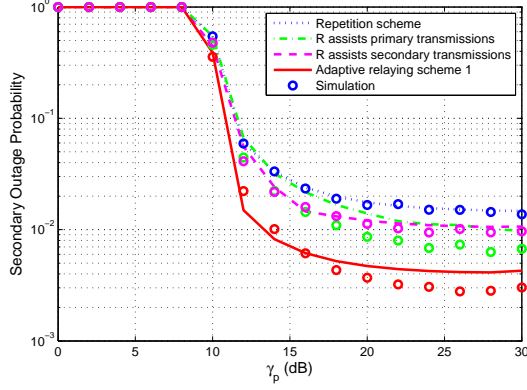


Fig. 2. Comparison of different relaying schemes

C. Extension to “Adaptive relaying scheme 2”

In this extension, we assume that R is equipped with a SIC (Successive Interference Cancellation) receiver [9], and hence it is able to decode both signals. We define $a_{ps} = \frac{(1-\alpha)\gamma_r^{(ps)}|h_{rs}|^2}{\alpha\gamma_r^{(ps)}|h_{rs}|^2+1}$, where $0 \leq \alpha \leq 1$. We also define $C_i = \{a_i = \max\{a_0, a_p, a_s, a_{ps}\}\}$ ($i = 0, p, s$ or ps) and $E = \{A_p \cap B_p\} \cup \{A_s \cap B_s\}$ the event of a successful successive decoding of both signals, where $B_i = \{\frac{1}{2}\log_2(1 + \gamma_i|h_{ir}|^2) \geq R_i\}$. We call this extension “Adaptive relaying scheme 2”. For this scheme, we distinguish four relaying procedures:

$$\begin{aligned} &\text{If } E \cap C_{ps}, \text{ then} & D = 3, \\ &\text{If } \{A_s \cap \bar{B}_s \cap (a_s > a_0)\} \cup \{E \cap C_s\}, \text{ then} & D = 2, \\ &\text{If } \{A_p \cap \bar{B}_p \cap (a_p > a_0)\} \cup \{E \cap C_p\}, \text{ then} & D = 1, \\ &\text{Otherwise} & D = 0. \end{aligned}$$

The relaying procedures for $D = 0, 1$ or 2 are identical to the first scheme. When $D = 3$, a fraction of the relay transmit power $\alpha P_r^{(ps)}$ is used to send x_p and the rest, i.e., $(1-\alpha)P_r^{(ps)}$ is used to transmit x_s . SINRs at PD and at SD are then given by (eq.(10), [8]) and (eq.(11), [8]) respectively. The parameter α is calculated using (eq.(33), [8]) where $\gamma_r^{(ps)} = \gamma_r^{max}$.

IV. NUMERICAL AND SIMULATION RESULTS

We consider the CRN presented in Fig. 1 where the coordinates of PT, ST, PD and SD are given by (0,1.82), (0,0), (1,1.82) and (1,0) respectively (coordinates are in distance units). We assume that $R_p = 0.8 \text{ bits/s/Hz}$, $R_s = 0.2 \text{ bits/s/Hz}$, $\varepsilon = 0.1$, $\beta = 4$ and $\gamma_p = \gamma_r^{max} = 20 \text{ dB}$. We measure the average of the secondary outage probability calculated for different random positions of the relay node in the plan of coordinates (X,Y) where $0.1 \leq X \leq 0.9$ and $0.1 \leq Y \leq 1.7$ unless otherwise is stated.

In Fig. 2, we compare the “Adaptive relaying scheme 1” to other transmission schemes presented in the literature [6], [8]. At low γ_p ($\gamma_p \leq 8 \text{ dB}$), no secondary transmissions are allowed. When γ_p is higher than the cutoff value, the “Adaptive relaying scheme 1” presents, as expected, the best

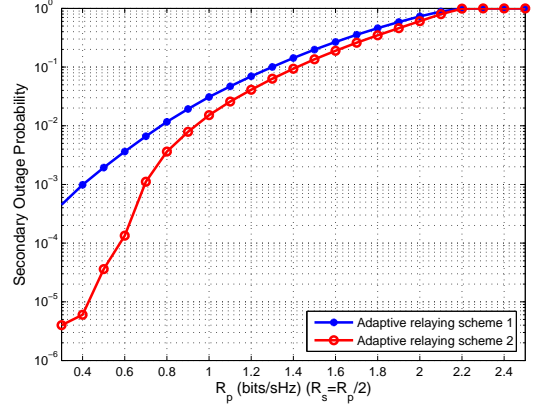


Fig. 3. “Adaptive relaying scheme” versus “Adaptive relaying scheme 2” (simulations only)

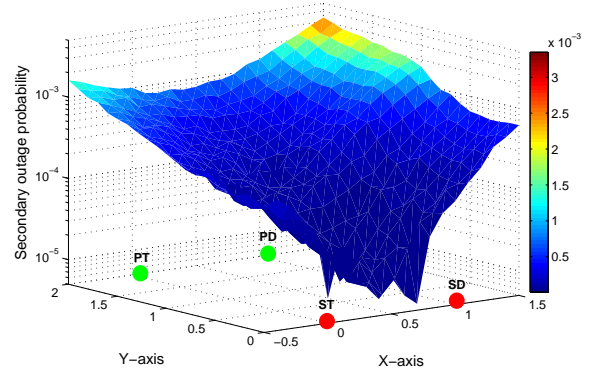


Fig. 4. Secondary Outage Probability vs. Relay position (simulations only)

outage performance since the proposed scheme chooses the most adequate relaying procedure. The “R assists secondary transmissions” and “R assists primary transmissions” schemes outperforms the non cooperative scheme. When R assists only the secondary transmissions, the performances are degraded by the fact that the relay transmit power is limited to P_r^{max} .

In Fig. 3, we compare the secondary outage probabilities of the two adaptive relaying schemes for different R_p values and where we assume $R_s = \frac{R_p}{2}$. For $R_p \leq 2.2 \text{ bits/s/Hz}$, the second scheme outperforms the first one. Indeed, the proposed second scheme offers more relaying possibilities for R and hence improves the secondary performance. For $R_p \geq 2.2 \text{ bits/s/Hz}$, no secondary or relaying transmissions are allowed due to the high primary outage probability requirement that blocks any interfering transmission.

In Fig. 4, we present the secondary outage performance using the “Adaptive relaying scheme 2” for different positions of the relay node on the plan (X,Y) where $-0.5 \leq X \leq 1.5$ and $0 \leq Y \leq 2$. When R is close to the secondary nodes, the secondary outage performance is improved. Indeed, when R is close to the primary nodes, the cases $D = 1$ and $D = 3$ are predominant and the outage probability gain comes from the interference reduction. As R gets closer to the secondary

nodes, the cases $D = 2$ and $D = 3$ occur more often. Hence, the outage performance gain is issued from the interference reduction ($D = 3$) and cooperation ($D = 2$). Moreover, when R is in the middle zone, the secondary outage probability decreases. In this zone, the condition of the channels linked to the relay node favors successful decoding and efficient forwarding of the signals.

V. CONCLUSION

In this paper, we proposed adaptive relaying schemes for cognitive radio networks, where a relay node is able to decide assisting the primary, the secondary or both transmissions depending on the channels condition. We showed by analysis and by simulation that the first adaptive relaying scheme, where the relay may help the primary or the secondary transmission, outperforms the non adaptive relaying schemes in terms of secondary outage probability, with respect to a primary outage probability threshold. Then, a second scheme has been proposed considering that the relay may help simultaneously both transmissions. Simulations show the performance improvement of the second scheme, specially at low data rates.

APPENDIX A PROOF OF EQ.(9)

Due to the independency between events, we have:

$$\begin{aligned} P(D = 1) &= P(A_p \cap \bar{A}_s) P(a_p > a_0) \\ &+ P(A_p | A_s) P(A_s) P(E_p) \\ &= P(\gamma_p | h_{pr}|^2 \geq \max\{\Lambda_p(1 + \gamma_s | h_{sr}|^2), \\ &\quad \frac{\gamma_s | h_{sr}|^2}{\Lambda_s} - 1\}) P(\gamma_r^{(p)} | h_{rs}|^2 \leq \gamma_p | h_{ps}|^2) \\ &+ P(\gamma_p | h_{pr}|^2 \geq \frac{\Lambda_p(1 + \Lambda_s)}{1 - \Lambda_p \Lambda_s}) P(a_p > a_0) \\ &P(\frac{\gamma_s | h_{sr}|^2}{\gamma_p | h_{pr}|^2 + 1} \geq \Lambda_s) P(a_p > a_s). \end{aligned} \quad (16)$$

Since $\gamma_a | h_{ab}|^2$ and $\gamma_c | h_{cb}|^2$ are exponential distributed random variables with parameters $1/\tilde{\gamma}_{ab}$ and $1/\tilde{\gamma}_{cb}$ respectively ($a = p$ or s , $c = p$ or s , and $b = p$, s or r), then $\forall x \in \mathbf{R}$:

$$P(\gamma_a | h_{ab}|^2 \leq \gamma_c | h_{cb}|^2) = \frac{\tilde{\gamma}_{cb}}{\tilde{\gamma}_{ab} + \tilde{\gamma}_{cb}}, \quad (17)$$

$$P(\gamma_a | h_{ab}|^2 \geq (\gamma_c | h_{cb}|^2 + 1)x) = \frac{\tilde{\gamma}_{ab} e^{-\frac{x}{\tilde{\gamma}_{ab}}}}{\tilde{\gamma}_{ab} + x \tilde{\gamma}_{cb}}, \quad (18)$$

and the probability density function (pdf) of $X_{ab} = \frac{\gamma_a | h_{as}|^2}{\gamma_b | h_{bs}|^2 + 1}$ ($a = s$ or r ; $b = p$ or r) is given by:

$$f_{X_{ab}}(x) = \frac{e^{-\frac{x}{\tilde{\gamma}_{as}}}}{\tilde{\gamma}_{as} + x \tilde{\gamma}_{bs}} \left(1 + \frac{\tilde{\gamma}_{as} \tilde{\gamma}_{bs}}{\tilde{\gamma}_{as} + x \tilde{\gamma}_{bs}} \right), \quad x \geq 0. \quad (19)$$

Since $\frac{1 + \frac{\tilde{\gamma}_{rs}^{(s)} \tilde{\gamma}_{ps}}{\tilde{\gamma}_{rs}^{(s)} + y \tilde{\gamma}_{ps}}}{\tilde{\gamma}_{ss} + y \tilde{\gamma}_{rs}^{(p)}} \leq \frac{1 + \tilde{\gamma}_{ps}}{\tilde{\gamma}_{ss}}$, $\forall y \geq 0$ and using (19), we get:

$$P(a_p > a_s) \leq -\phi_i \left(-\frac{1}{\tilde{\gamma}_{ps}} - \frac{\tilde{\gamma}_{rs}^{(s)}}{\tilde{\gamma}_{ss} \tilde{\gamma}_{ps}} \right) \left(\frac{1 + \tilde{\gamma}_{ps}}{\tilde{\gamma}_{ss}} \right) e^{\frac{1}{\tilde{\gamma}_{ps}} + \frac{\tilde{\gamma}_{rs}^{(s)}}{\tilde{\gamma}_{ss} \tilde{\gamma}_{ps}}}. \quad (20)$$

Using (17),(18) and (20) in (16), we obtain (9).

APPENDIX B PROOF OF LEMMA.1

From (10), ω , ω_1 and ω_2 are exponential random variables with parameters $1/\tilde{\gamma}_{rp}$, $1/\tilde{\gamma}_{sp}$ and $1/\tilde{\gamma}_{pp}$ respectively. Thus, the pdfs of ω and $\phi = \omega_2 - \Lambda_p \omega_1$ are:

$$f_\omega(\omega) = \frac{e^{-\frac{\omega}{\tilde{\gamma}_{rp}}}}{\tilde{\gamma}_{rp}}, \omega \geq 0; \quad f_\phi(\phi) = \begin{cases} \frac{e^{-\frac{\phi}{\tilde{\gamma}_{pp}}}}{\tilde{\gamma}_{pp} + \Lambda_p \tilde{\gamma}_{sp}}, & \phi \geq 0 \\ \frac{e^{-\frac{\phi}{\tilde{\gamma}_{pp}}}}{\tilde{\gamma}_{pp} + \Lambda_p \tilde{\gamma}_{sp}}, & \phi \leq 0. \end{cases}$$

Finally, we obtain the conditional primary outage probability:

$$\begin{aligned} P(z < \Lambda_p) &= \int_{-\infty}^{\Lambda_p} f_z(z) dz = \int_{-\infty}^0 \frac{\Lambda_p \tilde{\gamma}_{sp} e^{\frac{z}{\tilde{\gamma}_{pp}}}}{\beta_1 \beta_2} dz \\ &+ \int_0^{\Lambda_p} \frac{\Lambda_p \tilde{\gamma}_{sp} e^{-\frac{z}{\tilde{\gamma}_{rp}}}}{\beta_1 \beta_2} + \frac{\tilde{\gamma}_{pp} \left(e^{-\frac{z}{\tilde{\gamma}_{rp}}} - e^{-\frac{z}{\tilde{\gamma}_{pp}}} \right)}{\beta_1 (\tilde{\gamma}_{rp}^{(p)} - \tilde{\gamma}_{pp})} dz \\ &= \lambda_1 + \lambda_2, \end{aligned} \quad (21)$$

where $z = \omega + \phi$, $f_z(z)$ is its pdf, $\beta_1 = \tilde{\gamma}_{pp} + \Lambda_p \tilde{\gamma}_{sp}$, $\beta_2 = \tilde{\gamma}_{rp}^{(p)} + \Lambda_p \tilde{\gamma}_{sp}$; λ_1 and λ_2 are given by (12) and (13) respectively. This completes the proof of Lemma.1.

APPENDIX C PROOF OF LEMMA.2

Using (19) for X_{sb} ($b = p$ or r), we obtain:

$$\begin{aligned} P(X_{sp} + X_{sr} < \Lambda_s) &= \\ &= \int_0^{\Lambda_s} \frac{\left(1 + \frac{\tilde{\gamma}_{ss} \tilde{\gamma}_{rs}^{(p)}}{\tilde{\gamma}_{ss} + x \tilde{\gamma}_{rs}^{(p)}} \right)}{\tilde{\gamma}_{ss} + x \tilde{\gamma}_{rs}^{(p)}} \left(e^{-\frac{x}{\tilde{\gamma}_{ss}}} - \frac{\tilde{\gamma}_{ss} e^{-\frac{\Lambda_s}{\tilde{\gamma}_{ss}}}}{\tilde{\gamma}_{ss} + (\Lambda_s - x) \tilde{\gamma}_{ps}} \right) dx. \end{aligned} \quad (22)$$

Since $\frac{\left(1 + \frac{\tilde{\gamma}_{ss} \tilde{\gamma}_{rs}^{(p)}}{\tilde{\gamma}_{ss} + x \tilde{\gamma}_{rs}^{(p)}} \right)}{\tilde{\gamma}_{ss} + x \tilde{\gamma}_{rs}^{(p)}} \leq \frac{\tilde{\gamma}_{rs}^{(p)} + 1}{\tilde{\gamma}_{ss}}$, then using this upper bound in (22), we obtain (14). This completes the proof of Lemma.2.

REFERENCES

- [1] J. Mitola and G. Q. J. Maguire, "Cognitive radio: making software radios more personal," *IEEE Personal Commun.*, vol. 6, no. 4, pp. 13–18, aug. 1999.
- [2] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.*, vol. 23, no. 2, pp. 201–220, feb. 2005.
- [3] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inf. Theory*, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
- [4] W. Jaafar, W. Ajib, and D. Haccoun, "On the performance of multi-hop wireless relay networks," *Wireless Communications and Mobile Computing*, Dec. 2011, [Online] Available: <http://www.wileyonlinelibrary.com>.
- [5] Y. Han, A. Pandharipande, and S. Ting, "Cooperative decode-and-forward relaying for secondary spectrum access," *IEEE Trans. Wireless Commun.*, vol. 8, no. 10, pp. 4945–4950, october 2009.
- [6] Y. Zou, J. Zhu, B. Zheng, and Y.-D. Yao, "An adaptive cooperation diversity scheme with best-relay selection in cognitive radio networks," *IEEE Trans. Signal Process.*, vol. 58, no. 10, pp. 5438–5445, oct. 2010.
- [7] K. Lee and A. Yener, "Outage performance of cognitive wireless relay networks," in *IEEE Global Telecommun. Conf.*, dec. 2006, pp. 1–5.
- [8] W. Jaafar, W. Ajib, and D. Haccoun, "A novel relay-aided transmission scheme in cognitive radio networks," in *IEEE Global Telecommun. Conf.*, dec. 2011. [Online]. Available: <http://arxiv.org/abs/1109.2843v1>
- [9] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*. Cambridge University Press., 2005.